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On constitutive parameters for the anisotropic and compressible ionospheric plasma

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Abstract : Coupled wave equations are obtained in the study of radio wave propagation through the compressible and anisotropic plasma appropriate for the lower region of the ionosphere. A formal solution has been presented for the transverse components of the electric field within such a medium.

Keywords : Ionospheric anisotropy, ionospheric compressivity.

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1. Introduction

The ionosphere may be considered as a lossy, compressible plasma medium which is anisotropic under the influence of the geomagnetic field. The physical characteristics of an anisotropic medium are mostly governed by the Maxwell's field equations in conjunction with the hydrodynamic equation of motion and the combined equation of continuity and state [1–8].

During radio wave propagation through the lower ionosphere, a set of equations has been chosen from which coupled equations are obtained. Following a method [9], the governing differential equations for the anisotropic and compressible ionospheric plasma are decoupled in the analysis. A formal solution has been given for the transverse components of the electric field.

2. Mathematical formulation

The ionosphere has been taken as a partially ionised, electrically neutral and compressible medium traversed by the geomagnetic field, which introduces the anisotropy in the dielectric behaviour. The excitation is assumed to be time-harmonic and of small amplitude, such that all terms excepting the first-order perturbation are negligible in the various equations. The heavy ions and neutrals have been considered to be stationary. The dynamical behaviour of the partially ionised ionospheric plasma can be described by Navier-Stoke's equation of hydrodynamics which expresses the acceleration of a fluid in terms of a pressure gradient force, a viscous force and the resultant of all external forces. Thus, for time-harmonic variations under small signal assumption and for a stationary plasma, the electric field, the magnetic field and the other parameters of the medium satisfy the following equations :

Maxwell's equations

$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}, \tag{1}$$

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} - Ne\boldsymbol{\vartheta}; \tag{2}$$

Hydrodynamic equation of motion

$$mN\left(\frac{D\boldsymbol{\vartheta}}{Dt} + \nu\boldsymbol{\vartheta}\right) = -\nabla p - eN(\boldsymbol{E} + \boldsymbol{\vartheta} \times \boldsymbol{H}_0) + \eta \nabla^2 \boldsymbol{\vartheta}; \tag{3}$$

Equation for the conservation of mass

$$\nabla \cdot (N\boldsymbol{\vartheta}) = -\frac{\partial N}{\partial t}; \quad \text{and} \tag{4}$$

Combined equation of continuity and state

$$a^2 mN \nabla \cdot \boldsymbol{\vartheta} = -\frac{\partial p}{\partial t}. \tag{5}$$

Here

$\boldsymbol{E}$ = induced electric field,

$\boldsymbol{H}$ = induced magnetic field,

m = mass of the electron,

e = charge of the electron,

N = electron number density,

$\boldsymbol{\vartheta}$ = velocity of the electron,

ν = collision frequency of the electrons with heavy particles,

p = atmospheric pressure,

η = coefficient of viscosity of the medium,

$\boldsymbol{H}_0$ = geomagnetic field,

a = velocity of sound in electron gas,

μ_0 = permeability of the vacuum,

ε_0 = electrical permittivity of the vacuum.

The scalar and vector products of the Fourier transformed result of eq. (3) with $\boldsymbol{H}_0$ yield ($\vartheta \cdot \boldsymbol{H}_0$) and ($\vartheta \times \boldsymbol{H}_0$), which on substitution in eq. (3), give the expression of ϑ. Using ϑ in eq. (2), one can deduce

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon\eta \left\{ (\bar{\bar{\varepsilon}}) \cdot \boldsymbol{E} - \left[\frac{\omega_p^2}{Ne\left(\omega^2 - \omega_c^2\right)} + \frac{\nu\omega_p^2\left(\omega^2 + \omega_c^2\right)}{Ne\omega^2\left(\omega^2 - \omega_c^2\right)} - \frac{2}{9}\frac{\eta\omega_c^2\left(\omega^2 + \omega_c^2\right)}{Ne\omega^2\left(\omega^2 - \omega_c^2\right)} \right] \left[(\bar{\bar{\chi}}) \cdot \nabla p \right] \right\}, \tag{6}$$

where

ω = angular frequency of the wave,

ω_p = angular plasma frequency of an electron $= \left(\dfrac{Ne^2}{m\varepsilon_0}\right)^{1/2}$,

ω_c = angular gyrofrequency of an electron $= \dfrac{eH_0}{m}$,

$(\bar{\bar{\varepsilon}})$ and $(\bar{\bar{\chi}})$ are the required dyadics, which are given by

$$(\bar{\bar{\varepsilon}}) = \left[1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} + \frac{\nu\omega_p^2\left(\omega^2 + \omega_c^2\right)}{\omega^2\left(\omega^2 - \omega_c^2\right)} - \frac{1}{3}\frac{\eta\omega_c^2\left(\omega^2 + \omega_c^2\right)}{\omega^2\left(\omega^2 - \omega_c^2\right)} \right] (\bar{\bar{I}}) + \left\{ \frac{\omega_p^2}{\omega^2\left(\omega^2 - \omega_c^2\right)} + \frac{\nu\omega_p^2\left(\omega^2 + \omega_c^2\right)}{\omega_c^2\left(\omega^2 - \omega_c^2\right)} \right\} \left[\omega_c\omega_c + j\omega(\bar{\bar{\varpi}}_c) \right] \tag{7}$$

and

$$(\bar{\bar{\chi}}) = (\bar{\bar{I}}) - \frac{1}{\omega^2}\left[1 - \nu\omega_c^2 - \tfrac{2}{9}\nu\omega^2 \right] \left[\omega_c\omega_c + j\omega(\bar{\bar{\varpi}}_c) \right]. \tag{8}$$

$(\bar{\bar{I}})$ is the unit dyadic and $(\bar{\bar{\varpi}}_c)$ is another dyadic which satisfies the identity

$$(\bar{\bar{\varpi}}_c) \cdot \boldsymbol{L} = \omega_c \times \boldsymbol{L}, \tag{9}$$

where $\boldsymbol{L}$ is a vector.

$(\bar{\bar{\chi}})$, the compressivity tensor, is taken as a constitutive parameter for the ionospheric compressible medium.

In the derivation, we have

$$\omega_c = \begin{pmatrix} 0 \\ 0 \\ \omega_c \end{pmatrix}, \quad \left(\overline{\overline{\omega_c\omega_c}}\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \omega_c^2 \end{pmatrix},$$

and

$$\left(\overline{\overline{\omega}}_c\right) = \begin{pmatrix} 0 & -\omega_c & 0 \\ \omega_c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

Substituting ω_c, $\left(\overline{\overline{\omega_c\omega_c}}\right)$ and $\left(\overline{\overline{\omega}}_c\right)$ from (10) in (7) and (8), we obtain

$$(\overline{\overline{\varepsilon}}) = \begin{pmatrix} \varepsilon_{11} & j\varepsilon_{12} & 0 \\ -j\varepsilon_{12} & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix} \tag{11}$$

and

$$(\overline{\overline{\chi}}) = \begin{pmatrix} 1 & j\chi_{12} & 0 \\ -j\chi_{12} & 1 & 0 \\ 0 & 0 & \chi_{33} \end{pmatrix}. \tag{12}$$

The elements of $(\overline{\overline{\varepsilon}})$ and $(\overline{\overline{\chi}})$ can be expressed accordingly.

3. Derivation of coupled equations

To represent the results in terms of matrices, an orthogonal, cylindrical co-ordinate system has been considered, with $H_0 = \hat{s}H_0$, where $\hat{s}$ is a unit vector along z-direction.

The transverse and longitudinal dyadic components of $(\overline{\overline{\varepsilon}})$ and $(\overline{\overline{\chi}})$ have been made useful through the matrix equation (6). Curl and divergence operations of (6) and use of (1) and (2) will lead to the following coupled wave equations :

$$\nabla_t^2 H_z + \left[\mu_0\varepsilon_0\left(\varepsilon_{11} - \varepsilon_{12}\chi_{12}\right)\right]H_z - \frac{e\omega\varepsilon_{12}}{ma^2}\left(1 - \frac{\omega_c\varepsilon_{11}}{\omega\,\varepsilon_{12}}\right)p = 0 \tag{13}$$

and

$$\nabla_t^2 p - \left(\frac{\varepsilon_0\omega_c\omega_p^2\varepsilon_{11}}{\omega\,a^2\varepsilon_{12}}\right)p + \mu_0 eN\omega_c H_z = 0. \tag{14}$$

Eqs. (13) and (14) can be solved suitably for H_z and p. A formal solution for the transverse component of E can be determined using eq. (9) as

$$E_t = \frac{1}{j\omega\varepsilon_0}\left(\bar{\bar{\varepsilon}}_t\right)^{-1}\cdot\left(\nabla_t \times \hat{s}H_z\right) - \left[\frac{\omega_p^2}{Ne\left(\omega^2-\omega_c^2\right)} + \frac{\nu\omega_p^2\left(\omega^2+\omega_c^2\right)}{Ne\omega^2\left(\omega^2-\omega_c^2\right)}\right.$$

$$\left. - \frac{2}{9}\,\frac{\eta\omega_c^2\left(\omega^2+\omega_c^2\right)}{Ne\omega^2\left(\omega^2-\omega_c^2\right)}\right]\left(\bar{\bar{\varepsilon}}_t\right)\cdot\left(\bar{\bar{\chi}}_t\right)\cdot\left(\nabla_t p\right), \tag{15}$$

∇_t denotes the 'del' operation in a transverse plane.

4. Discussion

For the compressible and anisotropic plasma as discussed above, the coupled eqs. (13) and (14) are obtained. The solution for the transverse component of the electric field is given by (15). In the derivation of the compressivity and dielectric tensor, the viscosity term and collision term have been taken into consideration unlike in the previous work. Inclusion of these parameters is important in the lower ionospheric height range.

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